Acceleration of free fall by means of the simple pendulum

Apparatus Pendulum bob (e.g. a metal sphere with a hook attached, or with a hole bored through its centre), cotton, stop-watch, metre scale, stand and clamp, small improvised vice (e.g. two small metal plates—see Method).

Method Tie a two-metre length of the cotton to the pendulum bob and suspend the cotton from the jaws of an improvised vice, such as two small metal plates held in a clamp. Alternatively two coins, two halves of a cork split lengthwise, or the jaws of a pair of pliers serve equally well for the point of suspension when gripped in a clamp.

Place a piece of paper with a vertical mark on it behind the pendulum so that when the latter is at rest it hides the vertical mark from an observer standing in front of the pendulum.

Set the pendulum bob swinging through a small arc of about 10°. With a stop-watch measure the time for 20 complete oscillations, setting the watch going when the pendulum passes the vertical mark and stopping it 20 complete oscillations later when it passes the mark in the same direction. Repeat the timing and record both times.

Measure the length L of the cotton from the point of suspension to the point of attachment to the bob.

Shorten the length of the pendulum by successive amounts of about 10 cm by pulling the cotton through the vice and for each new length take two observations of the time for 20 oscillations.

Readings

Length of pendulum L/m	Time for 20 oscillations			Time for 1 oscillation (periodic time) T/s	T^2/s^2
	t_1/s	t_2/s	Mean t/s		

Plot a graph with values of T^2/s^2 as ordinates against the corresponding values of L/m as abscissae.

Experimental details

1. When counting the oscillations remember to say 'nought' when the stop-watch is started, for if you start at 'one' and stop at 'fifty', only 49 oscillations will have been timed.

2. Be careful to count *complete oscillations* and not 'swings' which are only half a complete oscillation.

3. Do not reduce the length of the pendulum below 50 cm as the experiment becomes increasingly inaccurate the shorter the length of the pendulum.

4. Should the oscillations of the pendulum bob become elliptical at any time the timing should be rejected, the pendulum stopped and set oscillating again and a new timing made.

Theory and calculation

The periodic time T of a simple pendulum l is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where g is the acceleration of free fall.

Since, in this experiment

$$l = L+\varepsilon$$

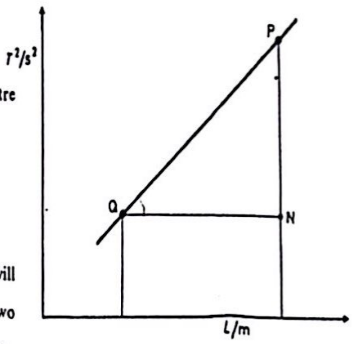
where ε is the extra constant length to the centre of gravity of the bob,

$$\therefore T = 2\pi\sqrt{\frac{L+\varepsilon}{g}}$$

and

$$T^2 = \frac{4\pi^2}{g}\cdot L + \frac{4\pi^2}{g}\cdot\varepsilon$$

from which it is seen that the graph of T^2 against L will be a straight line whose slope $\frac{PN}{QN}$, measured from two convenient and well-separated points P and Q on the line, is numerically equal to $\frac{4\pi^2}{g}$.



Thus

$$\frac{PN\,s^2}{QN\,m} = \frac{4\pi^2}{g}$$

$$\therefore g = 4\pi^2\left(\frac{QN}{PN}\right)\,m\,s^{-2}$$

Errors and accuracy

Errors in timing occur both when the stop-watch is started and when it is stopped. These errors are unlikely to be less than the interval at which the seconds hand moves (e.g. 0·1 or 0·2 s).

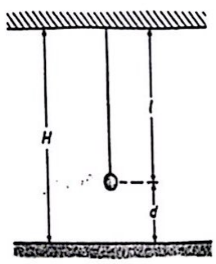
The error in the measurement of L is the error inherent in the use of any scale (half the distance between adjacent markings, doubled because of two ends to the distance measured).

As the value for g is obtained solely from the slope of the graph it follows that the % error in g is the same as the % error in the slope. Estimate the difference between the slope of your chosen 'best' straight line through the points and the slope of other possible straight lines drawn through the points and express this as a percentage (as described on p. 11).

State your value for g accordingly.

Additional experiment

If the experiment is performed with the pendulum suspended from an inaccessible point (e.g. the ceiling), and the height d of the bob above the floor varied and measured, both g and the height H of the point of suspension above the floor may be deduced.



Thus, since $l = H-d$ $\quad\therefore T = 2\pi\sqrt{\frac{H-d}{g}}$

whence

$$d = -\frac{g}{4\pi^2}\cdot T^2 + H$$

The negative slope of the graph of d against T^2 thus enables g to be evaluated and the intercept on the d-axis is the magnitude of H.

2 Experiments with a spiral spring